Determinanți. Inversa unei matrice

1. Să se calculeze determinanţii:

$$(a)\begin{vmatrix} 1 & 0 & 0 & -1 \\ 2 & 3 & 4 & 7 \\ -3 & 4 & 5 & 9 \\ -4 & -5 & 6 & 1 \end{vmatrix};\quad (b)\begin{vmatrix} 3 & -1 & 5 & 2 \\ 2 & 0 & 7 & 0 \\ -3 & 1 & 2 & 0 \\ 5 & -4 & 1 & 2 \end{vmatrix};\quad (c)\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 2 & 3 & 4 \\ 0 & 2 & 1 & 2 & 3 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 2 & 1 \end{vmatrix}.$$

$$(d)\begin{vmatrix} 1! & 2! & 3! \\ 2! & 2! & 3! \\ 3! & 3! & 3! \end{vmatrix};\ (e)\begin{vmatrix} 1! & 2! & 3! & 4! \\ 2! & 2! & 3! & 4! \\ 3! & 3! & 3! & 4! \\ 4! & 4! & 4! & 4! \end{vmatrix}$$

$$(f)\begin{vmatrix} 1 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 3 \end{vmatrix};\ (g)\begin{vmatrix} 1 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 4 \end{vmatrix}.$$

2. Să se calculeze determinanţii:

$$V_3(a,b,c)=\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix},\qquad V_4(a_1,a_2,a_3,a_4)=\begin{vmatrix} 1 & 1 & 1 & 1 \\ a_1 & a_2 & a_3 & a_4 \\ a_1^2 & a_2^2 & a_3^2 & a_4^2 \\ a_1^3 & a_2^3 & a_3^3 & a_4^3 \end{vmatrix},$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 3\alpha & 2\alpha+\beta & \alpha+2\beta & 3\beta \\ 3\alpha^2 & \alpha^2+2\alpha\beta & 2\alpha\beta+\beta^2 & 3\beta^2 \\ \alpha^3 & \alpha^2\beta & \alpha\beta^2 & \beta^3 \end{vmatrix},\ \alpha,\beta\in\mathbb{R},$$

$$D_n=\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ -1 & 2 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 2 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & -1 & 2 \end{vmatrix},\ n\geq 2.$$

3. Să se determine valorile parametrului $m\in\mathbb{R}$ astfel încât matricea $A=\begin{pmatrix} 1 & 0 & 1 \\ x & 1 & 2 \\ 1 & x & m \end{pmatrix}$ să fie inversabilă pentru orice $x\in\mathbb{R}$.

4. Să se calculeze inversele următoarelor matrice:

$$(a)\ A=\begin{pmatrix} 2 & 3 & 4 \\ 0 & 1 & 1 \\ 2 & 2 & -1 \end{pmatrix};\quad (b)\ B=\begin{pmatrix} 2 & 4 & 6 \\ 4 & 2 & 8 \\ 1 & 3 & 5 \end{pmatrix};$$

$(c)\ C = \begin{pmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & -2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$ $(d)\ D = \begin{pmatrix} 2 & 3 & 4 & 5 \\ 3 & 3 & 4 & 5 \\ 4 & 4 & 4 & 5 \\ 5 & 5 & 5 & 5 \end{pmatrix}$;

$(e)\ E = -\frac{1}{8}\begin{pmatrix} -3 & 11 & -1 \\ 2 & -10 & -2 \\ -2 & 2 & 2 \end{pmatrix}$; $(f)\ F = -\frac{1}{16}\begin{pmatrix} -14 & -2 & 20 \\ -12 & 4 & 8 \\ 10 & -2 & -12 \end{pmatrix}$;

$(g)\ G = \begin{pmatrix} 1 & 1 & -2 & -4 \\ 0 & 1 & 0 & -1 \\ -1 & -1 & 3 & 6 \\ 2 & 1 & -6 & -10 \end{pmatrix}$; $(h)\ H = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -4/5 \end{pmatrix}$.